Astronomical Enigma

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SOME RELATIONSHIPS CONNECTING THE SEMI-MAJOR AXES OF PLANETARY ORBITS.

Introduction

These notes give details of certain mathematical relationships connecting the lengths of semi-major axes of planetary orbits (assumed to be elliptical). Such relationships do not follow from known natural laws, being obtained by observation.

Throughout, equations are presented in exact form but, in most cases they relate to approximate quantities calculated from other approximate quantities or data.

Data Throughout, Astronomical Units (AU) are employed.

The lengths of the semi-major axes of the known planet's orbits are taken from Norton's Star Atlas. One is obliged to assume that the accuracy of the data is indicated by the number of decimal places given. Substituting the most recent NASA figures for Norton's data makes only a trivial difference to the resulting values. In most cases (e.g. Equation ..6) the results are even more accurate.

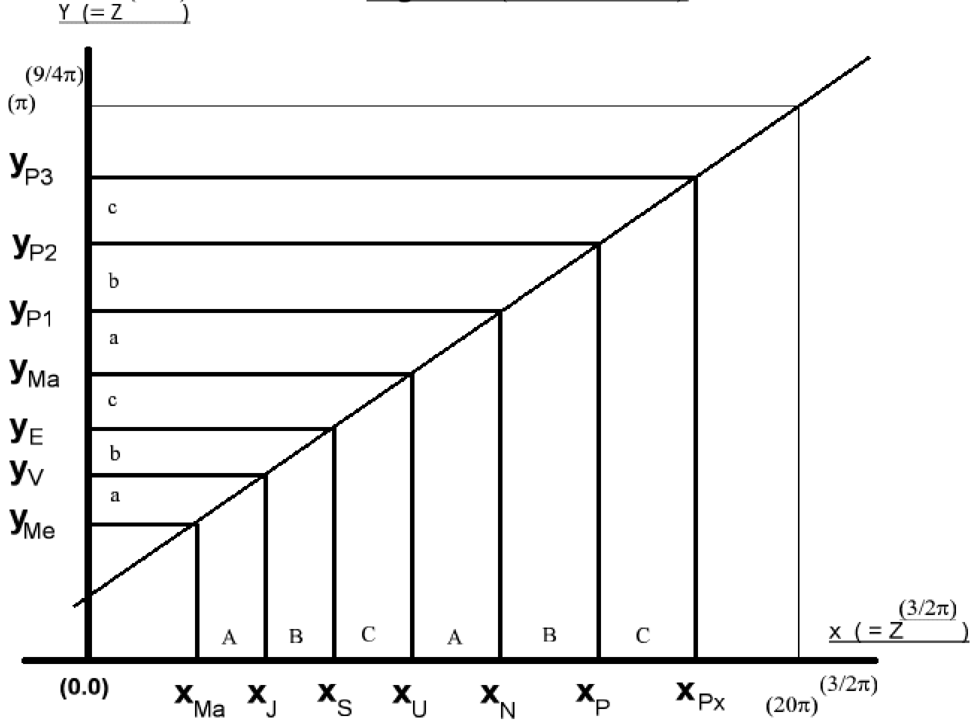
In the table below, semi-major planetary axes lengths, (the arithmetic mean of aphelion and perihelion distances) are denoted by Z;

Z-values have been converted into x and y values, where $x = Z^{(3/2\pi)}$ and $y = Z^{(9/4\pi)}$............. (1)

Planet	**Z value**	X value	Y value
Mercury (Me)	0.3870987		0.5067562
Venus (V)	0.7233322		0.7929725
Earth (E)	1.0000000		1.0000000
Mars (Ma)	1.5236915	1.2227197	1.3520425
Jupiter (J)	5.2028039	2.1977496	
Saturn (S)	9.5388437	2.9354505	
Uranus (U)	19.1818710	4.0976543	
Neptune (N)	30.0579240	5.0777748	
Pluto (P)	39.4390000	5.7809400	

Table 1

Figure 1 (Not to scale)



In figure 1, corresponding y-values for Mercury, Venus, Earth, and Mars are marked on the graph y-axis, and corresponding x-values for Mars, Jupiter, Saturn, Uranus, Neptune, and Pluto are marked on the x-axis. Additional points P1, P2, P3 and Px are also marked. (The graph is not drawn to scale)

The apparent near colinearity of the four points (X_{Ma}, y_{Me}), (X_J, y_V), (X_S, y_E) and (X_U, y_{Ma}) suggests that a good fitting linear equation in x and y may be determined.

Thus, consideration is given to a straight line of gradient **1/ln30** which passes through the point

$$((20\pi)^{3/2\pi},(\pi)^{9/4\pi}).$$

The equation of this line is $(y-(\pi)^{9/4\pi}) = (\ln 30)^{-1}(x-(20\pi)^{3/2\pi})$, which simplifies to :-

<u>y=0.2940141 x +0.1472341</u> .... (2)

Equation (2) is the first important finding of this investigation; the differences between known and equation calculated y-values for the four points are found to be very small, and consequently the corresponding pairs of Z-values also differ by small amounts.

The linear regression equation for the four points was found to be

<u>y=0.2933088 x +0.1464114</u> ....(3)

with correlation coefficient r = 0.999900, and from this, corresponding Z-values for Mercury, Venus, Earth and Mars were also calculated. The results obtained from both equations are shown in Table 2 with percentage differences (with respect to the data Z-values) being included.

Planet	Z (data)	Z (equ 2)	% difference	Z (equ 3)	% difference
Mercury	0.3870987	0.3870718	0.007	0.3852757	0.471
Venus	0.7233322	0.7238812	-0.076	0.7208603	0.342
Earth	1.000000	1.0144079	-1.441	1.0103543	-1.035
Mars	1.5236915	1.5236281	0.004	1.5177892	0.387

Table 2

It can be observed that in three of the four cases, the Z-values obtained from equation (2) are more accurate than those obtained by use of equation (3). This suggests that equation (2) may be employed with some confidence to predict other Z-values.

P1, P2, P3, and Planet X

Referring to Figure 1 and using the x-values for Neptune and Pluto in equation (2), we can now determine the y-values (and hence the corresponding Z-values) for P1 and P2.

Thus for P1, y = 1.6401715 and Z = 1.9954545 and for P2, y = 1.8469120 and Z = 2.3552069.

The first of these Z-values is very close to the nominal inner boundary of the asteroid belt, but P2 does not appear to be of any significance. However, if we suppose that a point P3 on the y-axis corresponds to the nominal outer boundary of the asteroid belt at Z = 3, then the corresponding y-value is $3^{(9/4\pi)}$ = 2.1964075

and, substituting this into equation (2) then gives us an x-value of 6.9696435 (marked as X_{px} on Figure 1). If this relates to an 'unknown', orbit Px, then the Z-value for this orbit is given by

$X_{px}{}^{(2\pi/3)}$= semi-major axis = 58.3466693 AU ....(4)

Interval Differences of x and y

With reference to Figure 1, let

$y_V - y_{Me}$ = a = 0.7929725 - 0.5067562 = 0.2862163

$y_E - y_V$ = b = 1.0 - 0.7929725 = 0.2070275

$y_{Ma} - y_E$ = c = 1.3520425 - 1.0 = 0.3520425

$y_{P1} - y_{Ma}$ = 1.6401715 - 1.3520425 = 0.2881290 = a (to within 0.67%)

$y_{P2} - y_{P1}$ = 1.8469120 - 1.6401715 = 0,2067405 = b (to within 0.14%)

$y_{P3} - y_{P2}$ = 2.1964075 - 1.8469120 = 0.3494956 = c (to within 0.73%)

This repetition of a, b, c, along the y-axis is another observation that has no ready explanation. Also the repeatability of the similar x-axis set of differences A,B,C, now follows, where
A/a = B/b = C/c = ln30 ....(5)

Through equation (1) several relationships exist between the x, y, and hence Z-values for pairs or groups of planets, for example, from equation (1)

y_{Me} = 0.2940141 x_{Ma} - 0.1472341 and

y_{Ma} = 0.2940141 x_{U} - 0.1472341

In terms of Z, these equations become $Z_{Me}^{(9/4\pi)}$ = 0.2940141 $Z_{Ma}^{(3/2\pi)}$ - 0.1472341 and

$Z_{Ma}^{(9/4\pi)}$ = 0.2940141 $Z_{U}^{(3/2\pi)}$ - 0.1472341

Eliminating the Z_{Ma} between them now gives an equation connecting Z_{Me} and Z_{U}; it follows that, by solving this equation for Z_{U}, we can determine Z_{U} from a known value of Z_{Me}.

Although this does not provide us with new information, since it is derived from the original fundamental equation (2), a similar observation can be applied to relationships which involve the repetitive characteristics of a,b,c or A,B,C. Orbits are interrelated by simple addition and/or subtraction of these values, in a variety of combinations.

However, in what follows, two additional relationships are determined which, apparently, do nor derive from previous results.

An Equation for Venus

From the previous section, b/a = 0.2070275 / 0.2862163 = 0.7233253, which is almost equal to Z_{V} (taken as 0.7233322) . This difference is less than 0.001%

However, b = [$Z_{E}^{(9/4\pi)}$ - $Z_{V}^{(9/4\pi)}$]

And a = [$Z_{V}^{(9/4\pi)}$ - $Z_{Me}^{(9/4\pi)}$]

Thus we may take

$$\mathbf{Z_{V} = [\, Z_{E}^{(9/4\pi)} - Z_{V}^{(9/4\pi)} \,] / [\, Z_{V}^{(9/4\pi)} - Z_{Me}^{(9/4\pi)} \,]} \qquad \text{.................. (6)}$$

This equation can be solved for Z_{V} using an iterative method, (Assuming Z_{V} is an unknown) and the result (0.723331002) is within 0.00017% of the figure for Venus mean orbit given by Norton's Star Atlas.

However, formation of the equation requires prior knowledge of Z_{V}, and it's relationship with Z_{E} and Z_{Me}.

Nevertheless, the equation is not predicted by theory, and is apparently unrelated to previous findings.

Pythagoras

Consider $a^2 + b^2 = (0.2862163)^2 + (0.2070275)^2 = 0.1247802$

And $c^2 = (0.35204525)^2 = 0.1239339$

Thus $a^2 + b^2 = c^2$ ....(7)

to an accuracy of better than 99.3%

It follows that $A^2 + B^2 = C^2$ ....(8)

These Pythagorean relationships (for a,b,c and A,B,C) could be illustrated by reference to corresponding right-angled triangles.

Kepler's Third Law

Since the semi-major axes (measured in astronomical units) of the planetary orbits convert to period by the application of Kepler's third law, it follows that in most of the equations the semi-major axis Z-values could be replaced by orbital period, in years, provided the exponent is changed accordingly.

$Z^{(9/4\pi)}$ is interchangeable with $P^{(3/2\pi)}$

And $Z^{(3/2\pi)}$ is interchangeable with $P^{(1/\pi)}$

(Where P is orbital period of respective planets in years)

Summary and conclusions.

It would seem that the orbits of the Solar System are interrelated in a number of different ways. This is an observed and verified fact that requires explanation.

We may reasonably conclude that the origin, laws, and dynamics of the Solar System are not yet fully understood.